Homework for CST-407-02P

Assigned 4/12/07,
Due 4/19/07

Frequency Response

1. Consider a pair of systems with transfer functions

$$H(z)=\frac{0.1465\,z}{z^2-1.74\,z+0.8865} \tag{1}$$

and

$$H(z)=\frac{0.0025\,z}{z^2-1.884\,z+0.8865} \tag{2}$$

a) Find the response of each system at DC, π/8, π/4, 3π/8, π/2, 3π/4, 7π/8, 5π/4, and -π/8.

```
-->H1 = 0.1465 * %z / (%z^2 - 1.74 * %z + 0.8865); H1.dt = 'd'
 H1  =


        0.1465z
   -----------------
                   2
   0.8865 - 1.74z + z

-->H2 = 0.0025 * %z / (%z^2 - 1.884 * %z + 0.8865); H2.dt = 'd'
 H2  =


         0.0025z
   -------------------
                    2
   0.8865 - 1.884z + z

-->w = %pi * [0 1/8 1/4 3/8 1/2 3/4 7/8 5/4 -1/8]
 w  =


        column 1 to 5
```

```
    0.     0.3926991    0.7853982    1.1780972    1.5707963

         column 6 to 9

    2.3561945    2.7488936    3.9269908  - 0.3926991

-->format(18)  // to dodge a Scilab display bug


-->[w'/%pi horner(H1, exp(%i * w'))] // show frequency vs.
                                     // response
 ans  =

    0        1.
    0.125    0.224101889715910 - 3.357933294845063i
    0.25   - 0.347233567171729 - 0.068632603944013i
    0.375  - 0.142389465053141 - 0.014666024457172i
    0.5    - 0.083838673947200 - 0.005468787064947i
    0.75   - 0.047625977833189 - 0.001243461170638i
    0.875  - 0.042056123567000 - 0.000524473940367i
    1.25   - 0.047625977833189 + 0.001243461170638i
  - 0.125    0.224101889715910 + 3.357933294845063i

-->[w'/%pi horner(H2, exp(%i * w'))]
 ans  =

    0        0.999999999999977
    0.125  - 0.016184211967531 - 0.004981913493489i
    0.25   - 0.004450352321495 - 0.000649349589828i
    0.375  - 0.002133961790204 - 0.000192560149521i
    0.5    - 0.001322165294101 - 0.000079652739321i
    0.75   - 0.000776407507050 - 0.000019363789854i
    0.875  - 0.000689195284952 - 0.000008253580460i
    1.25   - 0.000776407507050 + 0.000019363789854i
```

```
- 0.125   - 0.016184211967531 + 0.004981913493489i
```

b) Compare the system's response at DC, π/8, and π/2. Describe any interesting differences.

They both have practically the same response at DC (theoretically it is exact).

At π/8 H1 shows a great deal of peaking, while H2 does not show any peaking.

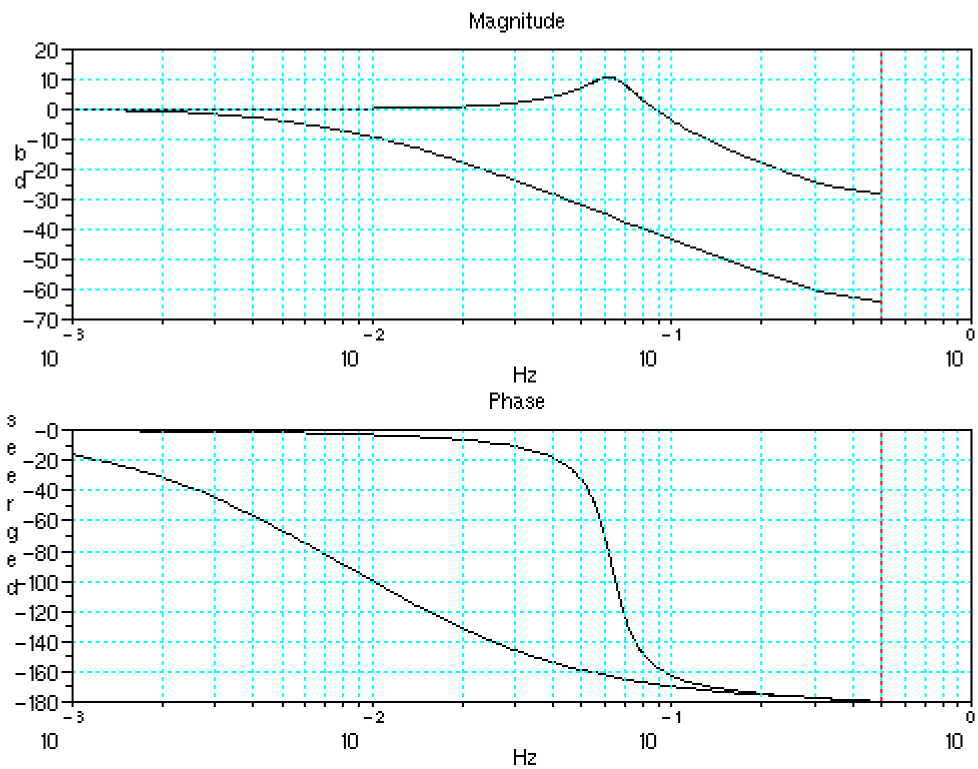
c) What is the same about each system's response at π/8 and -π/8? What is different? What about 3π/4, and 5π/4? What is special about these pairs of frequencies?

At π/8 and -π/8 the amplitudes are the same, as are the real portions of the responses. The imaginary parts are negatives of each other, making the two responses into complex conjugates. This is expected, as the exponentials of the frequencies are $e^{j\pi/8}$ and $e^{-j\pi/8}$, which are themselves complex conjugates of one another.

Because $e^{j5\pi/4} = e^{j\text{-}3\pi/4}$, the frequencies are essentially negatives of one another, and the discussion for frequencies equal to π/8 and -π/8 apply.

d) Use SciLab to generate Bode plots for the two systems. Compare and contrast the two responses.

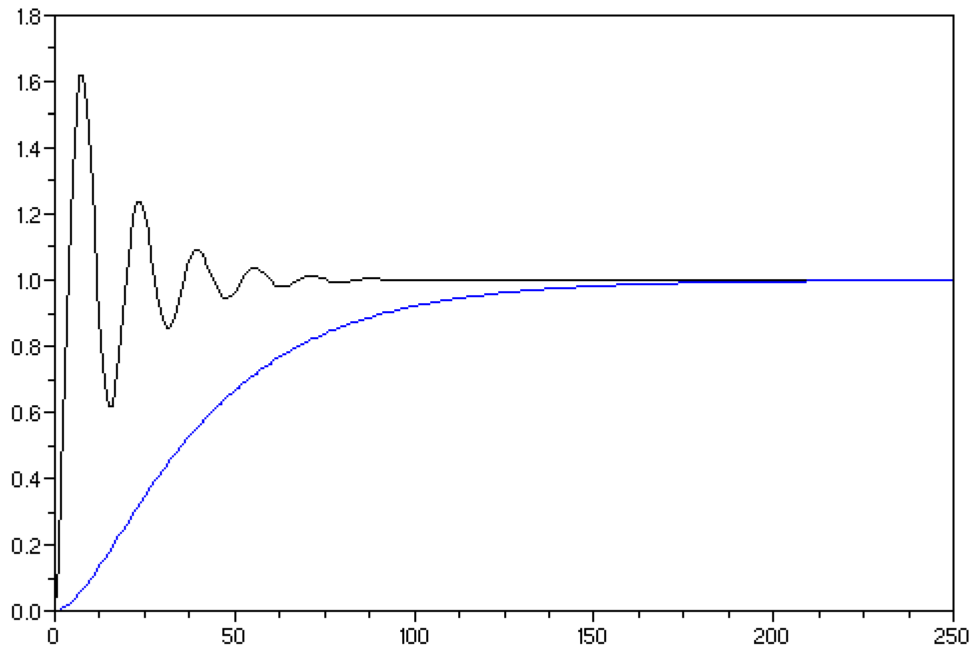
```
-->clf; bode(H1); bode(H2)
```



H1's response is very peaky, going up to 10dB above the DC value before dropping off. It's phase response is similarly sudden. H2's response is very slow, starting out low and going lower, with a slow phase response.

e) Use SciLab to generate the unit step response for the two systems. Compare and contrast the two responses.

```
-->k = 0:250;
-->h1 = flts(ones(k), H1); // 'ones(k)' just makes a matrix
-->h2 = flts(ones(k), H2); // of all ones the same size as k
-->clf; plot2d(k, [h1' h2']);
```



H1's step response (in black) rings like a bell, due to the underdamped nature of the system. H2 is very overdamped, and while it takes much longer to respond it never overshoots.

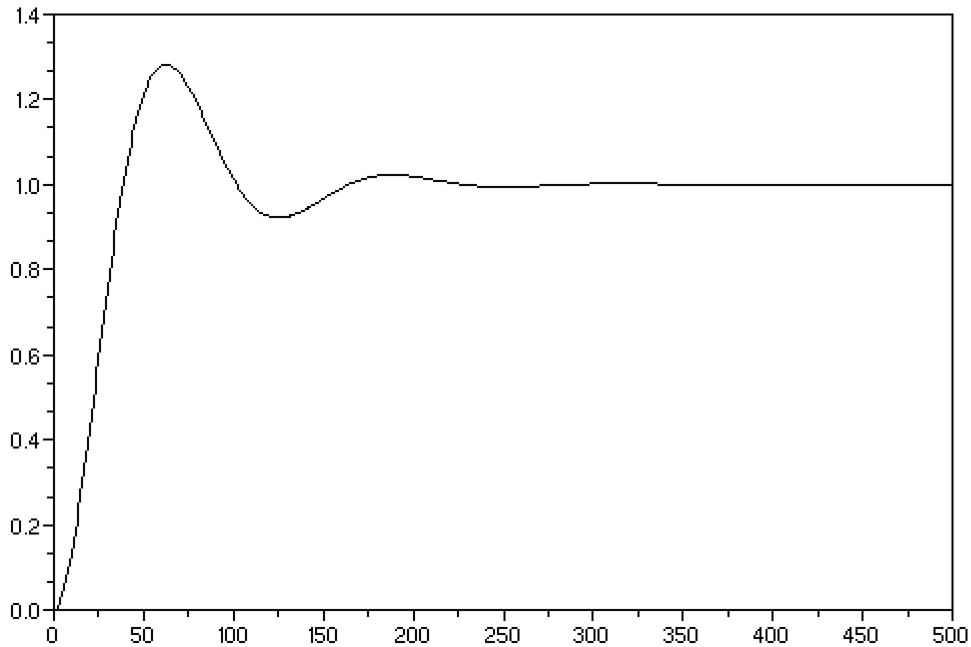
Note that H1 and H2 have close to the same natural frequency; note how differently they respond even so.

Performance

2. A system has a step response of

$$y_k = u(k)\left(1-(1.07)(0.98)^k \cos(0.05k-0.364)\right) \tag{3}$$

In SciLab, generate y for k ranging from 0 to 500. A plot of y should look like this:




```
-->k = 0:500;

-->y = 1 - 1.07 * (0.98 .^ k) .* (cos(0.05 * k - 0.364));

-->clf; plot2d(k, y);
```

a. Find the overshoot from the data (hint: use SciLab's 'max' operator)

```
-->max(y)
 ans  =

    1.2809661
```

b. Find the final value

```
-->y(501)
 ans  =

    0.9999614
```

c. If you type "min(find(y >= 0.1))" SciLab will report 10. This is the index of the first value of y that is greater than or equal to 0.1, and is also, the index of the delay time for y to reach 10% of its final value. You can find the actual delay time by typing k(10). What is the actual delay time, the value of y at this delay time, and the value of y at the previous sample?

```
-->k(10)
 ans  =

    9.

-->y(10)
 ans  =

    0.1111869

-->y(9)
 ans  =

    0.0902734
```

d. Use a procedure similar to the one above to find the rise time to within 5% of the final value.

```
-->min(find(y >= 0.95))
 ans  =

    38.
```

The rise time is 37 sample periods

e. Find the settling time to within 5% of the final value.

```
-->max(find(abs(1 - y) > 0.05))
 ans  =

    144.
```

The settling time is 143 sampling periods

3. A system is sampled at 200Hz, and has the transfer function

$$T(z)=\frac{0.03}{z-0.97} \quad . \tag{4}$$

a. What is the system's time constant?

$$\tau=\frac{1}{(-\ln 0.97)(200\,\mathrm{Hz})}\simeq 164\,\mathrm{ms}$$

b. What is the delay time to 10% of the final value?

$$t_d \simeq \frac{1}{200\,\mathrm{Hz}} \frac{\ln(1-0.1)}{\ln(0.97)} \simeq 17\,\mathrm{ms} \quad \text{(see equation 3.4)}$$

c. What is the rise time to within 10% of the final value?

$$t_d \simeq \frac{1}{200\,\mathrm{Hz}} \frac{\ln(1-0.9)}{\ln(0.97)} \simeq 378\,\mathrm{ms} \quad \text{(see equation 3.4)}$$

d. How much overshoot can we expect from this system?

None, it is a first-order system.

e. If we consider that a system that is within 1% of its final value is “settled all the way”, how long will this take for this system?

Strictly speaking, it will take 756ms (using a calculation similar to that for rise time). If we assume an even 5 time constants, it'll take 820ms.

4.

a. What is the natural frequency of the system from problem 2, whose step response is described in (3)?

This is the absolute value of the exponent of the pole pair. This can be found by finding the inverse transform of (3), however it can also be done by fitting (3) to (3.10) on page 47, in which case we find that a = -ln 0.98 and θ = 0.05. Then w_n can be found from (3.12) on page 47:

$$w_n = \sqrt{(\ln 0.98)^2 + 0.05^2} \simeq 0.0539$$

b. What is the damping ratio of the system?

Knowing a and w_n we need only plug them into (3.13) on page 48:

$$\zeta = \frac{a}{w_n} \simeq \frac{-\ln 0.98}{0.0539} \simeq 0.37$$

c. For a linear system, the percent of overshoot in its step response will always be the same, no matter how big the step command is. This is not the case for a nonlinear system – why?

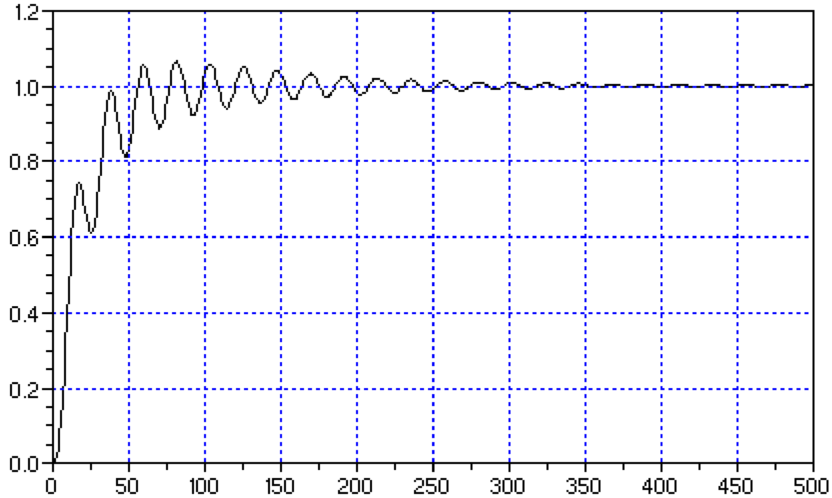
A linear system must obey superposition, which dictates that it's response to any step will be scaled by the same factor that the input is scaled to a unit step, thus insuring that overshoot will always be proportional. Nonlinear systems, by definition, don't follow superposition so this is not necessarily so.

Note that one could construct a perverse nonlinear system that *would* follow this rule, but would still be nonlinear.

d. A system has a rise time that is considerably shorter than it's settling time. You are told that the system “takes too long to stop moving”. All else being equal, which do you need to adjust first: its natural frequency or its damping ratio?

If the system's rise time is considerably shorter than it's settling time and it is a resonant system, then the problem is due to an underdamped pole. Addressing it's damping ratio first is the correct approach.

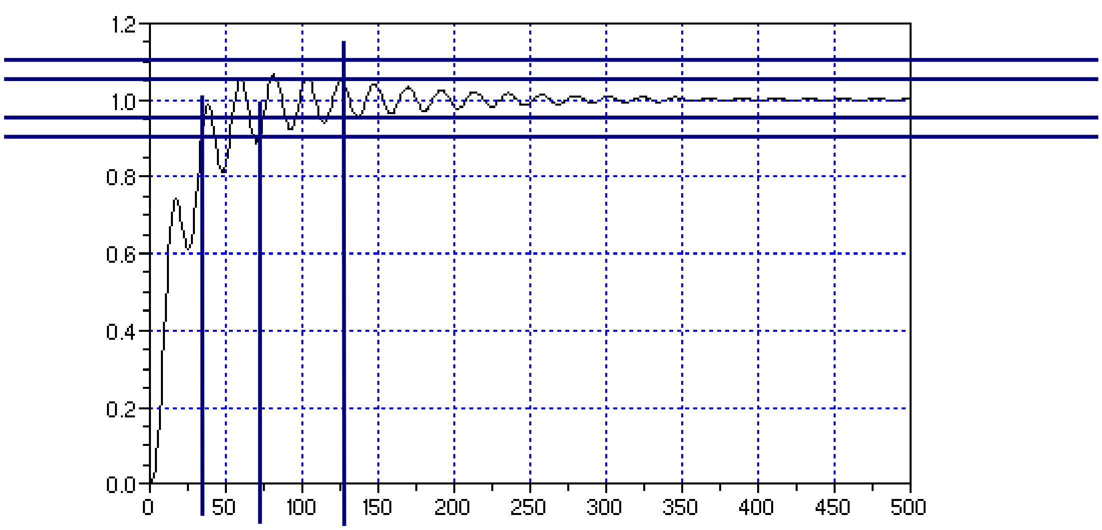
5. A system has the step response shown below, in amplitude vs. sampling ticks.



a. What is the minimum order that this system could be? Why?

b. Can this system be said to have a dominant pole? Why?

c. What is the rise time to within 5% of the final value?



It is impossible to be precise. The four horizontal traces are placed at approximately 90%, 95%, 105% and 110% of the final value; the points where the trace intersects with these lines will be our critical times.

The rise time to within 5% of the final value happens when the trace intersects the 95% line, or approximately 32 samples.

d. What is the settling time to within 10% of the final value?

   At the last time that the trace intersects the 90% or 110% line, which (as I read it) happens at approximately 49 samples.

e. What is the settling time to within 5% of the final value.

   At the last time that the trace intersects the 95% or 105% line, which happens at approximately 126 samples.